# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – APRIL 2010

**ST 2957 - RELIABILITY THEORY**

Date & Time: 23/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

**SECTION –A (10 x 2 = 20 marks)**

**Answer all the questions. Each question carries TWO marks**

1. Define the following: (a) Mean time before failure (MTBF).
   (b) Point availability.
2. If the hazard function r (t) = $3t^2$, t>0, obtain the corresponding probability distribution of time to failure.
3. In the usual notation, show tat MTBF = R*(0).
4. Explain in detail a parallel – series system of order (m,n).
5. What is meant by reliability allocation?
6. Define Coherent Structure and give two examples.
7. Define: (i) the number of critical path vectors of component i and (ii) relative importance of component i.
8. If $X_1$, $X_2$,…,Xn are associated binary random variables, show that $(1-X_1)$, $(1-X_2)$,…,(1-Xn) are also associated binary random variables.

9. Give an example of a set of random variables that are not associated.
10. Show that F is IFRA if and only if $\overline{F}(\alpha t) \geq \overline{F}^{\alpha}(t)$ for all 0<α<1 and t≥0.

**SECTION-B (5 × 8 = 40 marks)**

**Answer any FIVE questions. Each question carries EIGHT marks.**

11. Obtain the reliability function, hazard rate and the system MTBF for the following failure time density function.

    $$f(t) = 20 \exp(-5t^4)t^3, \quad t>0.$$

12. Find the system MTBF for a (k, n) system, when the lifetime distributions are independent exponential with the parameter λ. Assume that the components are non-repairable.

13. Suppose that $g_i(t)$ is the density function for $T_i$, the time to failure of $i^{th}$ component in a standby system with three independent components and is given by

    $$g_i(t) = \lambda_i e^{-\lambda_i t}, \; i=1,2,3; \; \lambda_1 \neq \lambda_2 \neq \lambda_3.$$

    Obtain the system failure time density function and hence find its expected value.

14. Find the mean life time of a (2, 3) system of independent components, when the component lifetimes are uniformly distributed on ( 0, i ), i =1,2,3.

15. Let Φ be a coherent structure. Show that $\Phi(\underline{x} \amalg \underline{y}) \geq \Phi(\underline{x}) \amalg \Phi(\underline{y})$.
    Further, show that the equality holds for all $\underline{x}$ and $\underline{y}$ if and only if the structure is parallel.

16. Let h be the reliability function of a coherent system. Show that

$$h(\underline{p} \amalg \underline{p'}) \geq h(\underline{p}) \amalg h(\underline{p'}) \quad \forall\ \underline{0} \leq \underline{p}, \underline{p'} \leq \underline{1}.$$

Also, show that the equality holds if and only if the system is parallel.

17. Suppose that $T_1, T_2, \ldots, T_n$ are random variables that are associated. Show that
(a) any subset of the associated random variables is associated.
(b) a set consisting of a single random variable is associated.

18. If the probability density function of F exists, show that F is an IFR distribution if and only if $r(t) \uparrow t$.

**SECTION-C (2 x 20 = 40 marks)**

**Answer any two questions. Each question carries TWENTY marks.**

19. (a) Obtain the reliability function and the system MTBF for Gamma failure time distribution with the parameters $\lambda$ and p. (10 marks)
(b) What is a (k, n) system? Obtain the system failure times density function for a (k, n) system, when the component failure times are independent and identically distributed. (2+8 marks)

20. A system consists of a single unit, whose lifetime X and repair time Y are independent random variables with probability density functions f (.) and g (.) respectively. Assume that initially at time zero, the unit just begins to operate.
(a). Determine the reliability and availability of the system. (2+6 marks)
(b) Show that $A_\infty = \dfrac{E(X)}{E(X)+E(Y)}$ (6 marks)
(c) If $f(t) = \lambda e^{-\lambda t}$, $\lambda > 0$, t > 0 and $g(t) = \mu e^{-\mu t}$, $\mu > 0$, t > 0, determine the reliability and availability of the system. (6 marks)

21. (a) Define: (i) Dual of a structure (ii) Path vector (iii) Cut vector (iv) Minimal path vector (v) Minimal cut vector. . (10 marks)
(b) Let h be the reliability function of a coherent system. Show that

$$h(\underline{p} \cdot \underline{p}') \leq h(\underline{p}) \cdot h(\underline{p}') \text{ for all } \underline{0} \leq \underline{p}, \underline{p}' \leq \underline{1}.$$ (10 marks)

22. (a) Examine whether Weibull distribution is a DFR distribution. Hence or otherwise, establish that exponential distribution is both IFR and DFR. (10 marks)
(b) State and prove IFRA closure theorem (10 marks)

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